Chain Rule

G) $y = \dfrac{x^2}{\sqrt{1+x^3}}$ $\quad y = \dfrac{x^2}{(1+x^3)^{1/2}}$

$y' = (1+x^3)^{1/2}\cdot(2x) - x^2\left[\frac{1}{2}(1+x^3)^{-1/2}\cdot 3x^2\right]$

$$y' = \frac{2x\sqrt{1+x^3} - \frac{3}{2}x^4(1+x^3)^{-1/2}}{1+x^3}$$

G) $y = \dfrac{x^2}{\sqrt{1+x^3}}$ $\quad y = x^2(1+x^3)^{-1/2}$

$\dfrac{dy}{dx} = x^2\left[-\frac{1}{2}(1+x^3)^{-3/2}\cdot 3x^2\right] + (1+x^3)^{-1/2}\cdot 2x$

$$= -\frac{3}{2}x^4(1+x^3)^{-3/2} + 2x(1+x^3)^{-1/2}$$

$$= \frac{-3x^4}{2\sqrt{(1+x^3)^3}} + \frac{2x}{\sqrt{(1+x^3)}}$$

H) $y = \left(5x + \sqrt[3]{x}\right)^4$

$\dfrac{dy}{dx} = -4\left(5x + x^{1/3}\right)^{-5}\cdot\left(5 + \frac{1}{3}x^{-2/3}\right)$

GCF

I) $y = x^4(3x-6)^5$

$\frac{d}{dx}(3x-6)^5$ $\frac{d}{dx}(3x-6)$

$y' = x^4\left[5(3x-6)^4 \cdot 3\right] + (3x-6)^5 \cdot 4x^3$

$= 15x^4(3x-6)^4 + 4x^3(3x-6)^5$ ✓

$= x^3(3x-6)^4\left[15x + 4(3x-6)\right]$ ✓

$= x^3(3x-6)^4\left[27x-24\right]$

J) $y = \frac{1}{(1-2x)^3}$

$y = \frac{1}{(1-2x)^3}$ $y = (1-2x)^{-3}$

$y' = -3(1-2x)^{-4} \cdot (-2)$

$y' = \frac{6}{(1-2x)^4}$

L) $y = \sqrt{3x\csc x}$

$y = (3x\csc(x))^{1/2}$

$y' = \frac{1}{2}(3x\csc x)^{-1/2} \cdot \left[3x(-\csc x\cot x) + \csc x(3)\right]$

M) $y = 3x\sqrt{\csc x}$

$y = 3x(\csc x)^{1/2}$

$y' = 3x\left[\frac{1}{2}(\csc x)^{-1/2} \cdot (-\csc x\cot x)\cdot 1\right] + (\csc x)^{1/2} \cdot 3$

2nd derivative

$9\cot\left(\frac{1}{3}x\right)$

N) Find y'' if $y = 9\cot\left(\frac{x}{3}\right)$

$y'' = 9\left(-\csc^2\left(\frac{x}{3}\right)\right)\cdot\left(\frac{1}{3}\right)$

$y' = -3\csc^2\left(\frac{1}{3}x\right)$

$y' = -3\left[\csc\left(\frac{1}{3}x\right)\right]^2$

$y'' = -6\left[\csc\left(\frac{1}{3}x\right)\right]\cdot\left(-\csc\left(\frac{1}{3}x\right)\cot\left(\frac{1}{3}x\right)\right)\cdot\frac{1}{3}$

$y'' = 2\csc^2\left(\frac{1}{3}x\right)\cot\left(\frac{1}{3}x\right)$